PERGAMON

International Journal of HEAT and MASS TRANSFER

International Journal of Heat and Mass Transfer 43 (2000) 1399–1408

www.elsevier.com/locate/ijhmt

Prediction of heat and mass transfer for fully developed turbulent fluid flow through tubes

S. Aravinth

Department of Chemical and Environmental Engineering, National University of Singapore, 119260 Singapore

Received 4 January 1999; received in revised form 25 June 1999

Abstract

A resistance-in-series model is developed to quantitatively predict the heat and mass transfer processes for turbulent fluid flow through tubes and circular conduits under uniform wall temperature condition. An analogy between heat, mass and momentum transfer has been suggested using Fanning number (*Fa*) for momentum transfer. A modified eddy diffusivity expression for the turbulent boundary layer near a smooth wall has been obtained from earlier models. This eddy diffusivity expression is used to predict radial temperature or concentration profile as well as film coefficients that are in good agreement with the experimental studies over wide range of Prandtl or Schmidt numbers. The proposed equations are compared with Chilton–Colburn analogy and other semi-theoretical models using wide range of experimental data.

Keywords: Turbulent flow; Transport phenomena; Resistance-in-series model

1. Introduction

Solid–liquid heat and mass transfer through tubes and conduits is of practical importance for the design and development of various unit operations. Many investigations have been done both theoretically and experimentally. In these previous studies, the experimental film coefficients were expressed in terms of dimensionless groups such as the Reynolds number (*Re*), Schmidt or Prandtl number (*Sc* or *Pr*). These equations can be represented as,

$$Nu = aRe^b\,Pr^c \tag{1}$$

However, significant differences in the values of *a*, *b*, and *c* have been observed. These discrepancies are mainly due to a lack of attention on hydrodynamic and thermal entry effects and on the fundamentals of transport process. The Colburn analogy recommends a value of 0.023, 0.80 and 0.33, while Dittus–Boelter equation gives 0.023, 0.80 and 0.4 for *a*, *b* and *c*. Gilliland and Sherwood [1] used the data of vaporization of liquids in a wetted wall column (a gas–liquid process) and obtained the value 0.44 for *c*, which is cited by most of the textbooks for gas–solid mass transfer. Recently, Dudukovic et al. [2] explained the discrepancy in the interpretation and supported the Colburn analogy.

In the past, few semi-theoretical models including the friction factor have also been proposed by many researchers. Table 1 lists the significant literature correlations of this kind. Some models [3–5] have been developed using approximate solutions for Lyon transport equation. A generalized formu-

E-mail address: s.aravinth@tech.chem.ethz.ch (S. Aravinth).

0017-9310/00/$ - see front matter
PII: S0017-9310(99)00218-5

Nomenclature

a, b, c	constants in Eq. (1)
A	constant in Eq. (14)
c_p	specific heat capacity (J/(kg K))
C^+	dimensionless concentration
D	diffusion coefficient (m^2 s^{-1})
d	diameter of the pipe (m)
f	fanning friction factor, dimensionless
F	function defined in Eq. (16), dimensionless temperature
h	heat transfer coefficient (J/(s m^2 K))
K	mass transfer coefficient (m s^{-1})
k	thermal conductivity (J/(s m K))
k_m	momentum transfer coefficient (m/s)
q	heat flux (J s^{-1} m^{-2})
r	radius of the pipe (m)
T	temperature (K)
T^+	dimensionless temperature
u	axial velocity (m s^{-1})
u^+	dimensionless velocity, $u/(\tau_w/\rho)^{1/2}$
u_b	average axial velocity (m s^{-1})
y	radial distance (m)
y^+	dimensionless distance from the wall, $y(\tau_w/\rho)^{1/2}/\nu$

Dimensionless

Fa	Fanning number, $k_m d/\nu$
Nu	Nusselt number, hd/k
Pr	Prandtl number, $c_p\mu/k$
Pr_t	turbulent Prandtl number, $\varepsilon_M/\varepsilon_H$
Re	Reynolds number, du_b/ν
Sc	Schmidt number, ν/D
Sh	Sherwood number, Kd/D
St	Stanton number, $Sh/(Re\ Sc)$, $Nu/(Re\ Pr)$

Greek symbols

α	molecular thermal diffusivity (m^2 s^{-1})
ε_H	eddy diffusivity of heat (m^2 s^{-1})
ε_M	eddy viscosity (m^2 s^{-1})
ν	kinematic viscosity (m^2 s^{-1})
ϕ	turbulent Prandtl number in turbulent core
ρ	density (kg/m^3)
τ	shear stress (kg m^{-1} s^{-2})

Subscripts

b	bulk
i	interface
t	turbulent layer
v-b	viscous-buffer zone
w	wall

lation of the analogy between heat and momentum transfer [6] was used by Friend and Metzner [7] to deduce a semi-theoretical correlation. Gnielinski [8] extended the equation of Petukhov and Popov [3] to higher Pr by modifying the exponent on Re. Pinczewski and Sideman [9] adapted the modified surface renewal concept assuming that the fluid renewal at the surface occurs infrequently. However, less information could be deduced regarding radial temperature or concentration profiles and Pr_t variation from these models. Previous researchers have assumed that only wall region controls the transfer process and the molecular diffusivity contribution to the transfer process is small and may be omitted in the buffer layer. Although this assumption is true at high Pr/Sc, it is no longer applicable for fully developed turbulent fluid flow over wide range of Pr/Sc.

The aim of the present work is to develop a new formulation for prediction of transfer processes in fully developed turbulent fluid flow through tubes using a simple and fundamental approach, similar to the model developed by Hughmark [10]. It is assumed that all the three layers namely; viscous sub-layer, buffer region, and the turbulent core offer resistance to the transfer process in series. The present analysis accounts for variation of heat or mass flux and turbulent Prandtl number in radial direction. A new eddy diffusivity expression of the form proposed by Notter and Sleicher [11] has been characterized for turbulent fluid flow near the wall with the parameters of Churchill [12]. This eddy diffusivity expression has been used in the present theory to predict temperature or concentration profiles and transfer coefficients. The analysis presented in this article is by relating momentum and energy fluxes and the results can be directly applied to mass transfer by replacing appropriate terms with analogous quantities.

2. Theory

A brief physical representation about the turbulent transport mechanism for a hot fluid flowing in a pipe with cooled walls is shown in Fig. 1. In case of turbulent fluid flowing past a solid surface, three layers namely the viscous sub-layer near the surface, buffer region and the turbulent layers exist. In the turbulent core the thermal energy is transported quickly from place to place by means of eddies, causing a small temperature difference. On the other hand, close to the

Table 1
List of significant literature correlations

Reference	Pr/Sc	Re	Correlation
Petukhov and Popov [3]	0.5–2000	$2300–5\times10^6$	$St=\frac{\frac{f}{2}\left(1-\frac{180}{Re^{0.75}}\right)}{1.07+12.7\sqrt{\frac{f}{2}}(Pr^{2/3})}-1$
Petukhov [4]	0.5–2000	$2300–5\times10^6$	$St=\frac{\frac{f}{2}}{1.07+12.7\sqrt{\frac{f}{2}}(Pr^{2/3})}-1$
Sandall et al. [5]	$0.7–10^6$	$10^4–2\times10^5$	$St=\sqrt{\frac{f}{2}}\left[12.48Pr^{2/3}-7.853Pr^{1/3}+3.613\ln(Pr)+5.8+2.78\ln\left(\frac{Re\sqrt{f/8}}{45}\right)\right]^{-1}$
Friend and Metzner [7]	0.5–3000	$10^4–10^6$	$St=\frac{\frac{f}{2}}{1.20+11.8\sqrt{\frac{f}{2}}(Pr-1)(Pr)^{-1/3}}$
Gnielinski [8]	$0.6–10^5$	$2300–10^6$	$St=\frac{\frac{f}{2}\left(1-\frac{1000}{Re}\right)}{1.07+12.7\sqrt{\frac{f}{2}}(Pr^{2/3})}-1$
Pinczewski and Sideman [9]	0.5–10	$2300–10^6$	$Sh=0.064\sqrt{\frac{f}{2}}ReSc^{1/2}(1.10+0.44Sc^{-1/3}-0.7Sc^{-1/6})$
	10–1000	$2300–10^6$	$Sh=0.0672\sqrt{\frac{f}{2}}Re\,Sc^{1/3}$

wall, in viscous sub-layer and buffer region the transport is by both molecular conduction and eddy diffusion. For convenience this region may be called as viscous-buffer zone (v-b zone). This results in wider temperature gradient in the region near the wall. Hence, based on the transport mechanism the radial plane can be divided into two regions, namely viscous-buffer zone and turbulent core separated by an interface. With this picture, the present model is developed with the following assumptions:

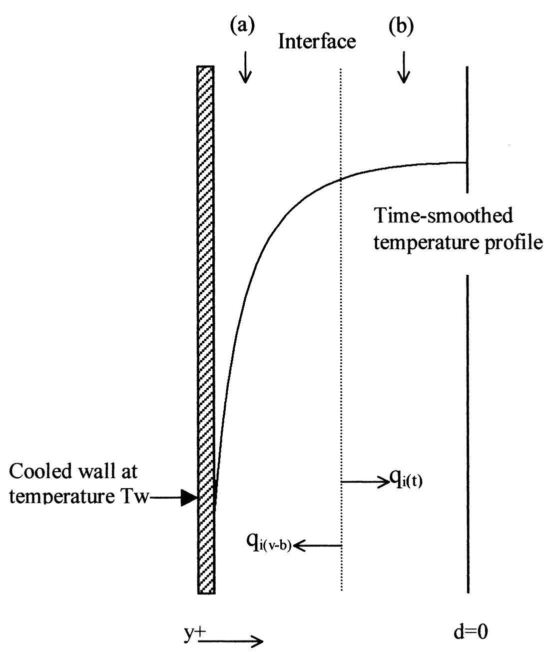

Fig. 1. Sketch showing temperature changes in (a) viscous-buffer zone and (b) turbulent core.

1. The fluid is Newtonian, incompressible, and has constant transport properties independent of time and position in the space. Previously developed one-dimensional models often assume invariable transport properties. During turbulent fluid flow this assumption is valid for most of the mass transfer studies and under small temperature gradient in heat transfer studies.
2. The turbulent flow through smooth tube is fully developed with no foam drag and entrance effects. The effect of body forces is small in comparison to that of viscous and inertial forces.
3. The axial velocity profile is independent of axial coordinate, similarly the temperature and concentration gradients.
4. The mass and energy fluxes vary in the radial direction. Previous works often assumed that flux varies linearly with respect to y,

$$\frac{q}{q_w}=\left(1-\frac{y}{r}\right) \tag{2}$$

A few others, following Reynolds analogy assumed constant heat flux in the radial direction,

$$q = q_{\mathrm{w}} \tag{3}$$

Earlier researchers found that the introduction of either assumption did not introduce great errors in representing the transfer processes. However, in the present work radial variation of mass or energy flux is taken into account by dividing the radial region into viscous-buffer zone and turbulent core. The flux at the interface is considered in radially opposite directions, as represented in Fig. 1 by following equations.

Flux at the interface towards the wall,

$$q_{i(\mathrm{v\text{-}b})} = h_{\mathrm{v\text{-}b}}(T_{\mathrm{i}} - T_{\mathrm{w}}) \tag{4}$$

Flux at the interface towards the axis,

$$q_{\mathrm{i(t)}} = -h_{\mathrm{t}}(T_{\mathrm{i}} - T_{\mathrm{b}}) \tag{5}$$

Although the fluxes in the opposite direction are practically impossible for fluid flow through pipe, Eqs. (4) and (5) are mathematically valid as the flux can be represented in the form of negative temperature gradient.

5. The shear stress is constant in the radial direction. Deissler and Eian [13] have found that the velocity, temperature, and concentration profiles are relatively insensitive to shear stress distribution for fully developed turbulent fluid flow, hence can be assumed constant.
6. The eddy diffusivity and eddy viscosity ratio varies along with the distance from the wall in the v-b zone and independent of *Pr*. Hence, the eddy diffusivity expressions suggested by Deissler [14] and many other researchers were not included in the present analysis as they essentially assume eddy diffusivity and eddy viscosity are equal or directly proportional, while the experimental observations [15] does not agree with this assumption. However, direct numerical simulations presented by Lyons et al. [16] suggest Pr_{t} to be unity at the wall. Hence, in the present case it is assumed that as $y^+ \rightarrow 0$, $Pr_{\mathrm{t}} = 1$.
7. The eddy diffusivity controls transfer process in viscous-buffer zone, while eddy viscosity controls the transport in turbulent core.

2.1. Development of model

In turbulent layer, the intensity of eddies will be higher than the v-b zone. Therefore, during the transfer process, the resistance to the exchange of heat between the bulk fluid and the solid surface is often confined to the viscous and buffer region. Although the resistance in turbulent core is smaller than v-b zone, it is appreciable at low *Pr*. Thus the total resistance to the transfer process is the sum of the resistances in the viscous-buffer zone and turbulent layer, represented by,

$$\frac{1}{h} = \frac{1}{h_{\mathrm{v\text{-}b}}} + \frac{1}{h_{\mathrm{t}}} \tag{6}$$

Eq. (6) represents a two-layer resistance separated by an interface, similar to that proposed by Hughmark [10]. The v-b zone and the turbulent core resistances in Eq. (6) are similar to the boundary layer resistance and the resistance corresponding to random eddies in the region beyond the boundary layer of Hughmark [10]. For a fully developed turbulent flow, the shear stress and the heat flux can be written as,

$$\tau = (\mu + \rho \varepsilon_{\mathrm{M}})\frac{\mathrm{d}u}{\mathrm{d}y} \tag{7}$$

$$q = \rho c_p(\alpha + \varepsilon_{\mathrm{H}})\frac{\mathrm{d}T}{\mathrm{d}y} \tag{8}$$

In the viscous-buffer region, Eq. (8) on integration with the boundary conditions: $T = T_{\mathrm{w}}$ at $y = 0$ and $T = T_{\mathrm{i}}$ at $y = y_{\mathrm{i}}$, and rearranging yields,

$$\frac{(T_{\mathrm{i}} - T_{\mathrm{w}})}{q_{\mathrm{i(v\text{-}b)}}} = \frac{1}{c_p\sqrt{\tau_{\mathrm{w}}\rho}}\int_0^{y_{\mathrm{i}}^+} \frac{\mathrm{d}y^+}{\dfrac{1}{Pr} + \dfrac{\varepsilon_{\mathrm{H}}(y^+)}{\nu}} \tag{9}$$

Combining Eqs. (4) and (9) yields,

$$\frac{1}{h_{\mathrm{v\text{-}b}}} = \frac{1}{c_p\sqrt{\tau_{\mathrm{w}}\rho}}\int_0^{y_{\mathrm{i}}^+} \frac{\mathrm{d}y^+}{\dfrac{1}{Pr} + \dfrac{\varepsilon_{\mathrm{H}}(y^+)}{\nu}} \tag{10}$$

As the transport is mainly by eddy means, the molecular diffusivity and the viscosity can be neglected in the turbulent core. Combining Eqs. (7) and (8) and integrating along the turbulent layer with the boundary conditions: $T = T_{\mathrm{b}}$ at $u = u_{\mathrm{b}}$ and $T = T_{\mathrm{i}}$ at $u = u_{\mathrm{i}}$, yields,

$$\frac{T_{\mathrm{b}} - T_{\mathrm{i}}}{u_{\mathrm{b}} - u_{\mathrm{i}}} = \frac{q_{\mathrm{i(t)}}}{c_p\tau_{\mathrm{w}}}\left(\frac{\varepsilon_{\mathrm{M}}}{\varepsilon_{\mathrm{H}}}\right)_{\mathrm{t}} \tag{11}$$

The velocity at the interface can be represented in terms of dimensionless velocity (u^+). With Eq. (5), Eq. (11) reduces to,

$$\frac{1}{h_{\mathrm{t}}} = \frac{u_{\mathrm{b}}\phi}{c_p\tau_{\mathrm{w}}}\left(1 - u_{\mathrm{i}}^+\sqrt{\frac{f}{2}}\right) \tag{12}$$

where ϕ represents the eddy viscosity to eddy diffusivity ratio in the turbulent core. Eqs. (10) and (12) can be added using the resistance in series definition (see Eq. (6)) and simplified to the following form,

$$St = \frac{\frac{f}{2}}{\phi + \sqrt{\frac{f}{2}}\left(\int_0^{y_i^+} \frac{dy^+}{\frac{1}{Pr} + \frac{\varepsilon_H(y^+)}{\nu}} - u_i^+\phi\right)} \tag{13}$$

The integral function in Eq. (13) can be evaluated using an accurate expression for eddy diffusivity in viscous sub-layer and buffer region. Extensive literature on eddy diffusivity relations has been reviewed by Notter and Sleicher [11]. Most of the previous works have been dealt with the assumption of Pr_t varying from 0.8 to 0.9, suggesting eddy mixing length of heat always greater than momentum mixing length. However, observed values [15] of Pr_t in the radial direction suggests that an eddy can carry heat to a lesser distance than momentum in the region near the wall. In effect, Pr_t can also be greater than unity. In this work the expression of the form suggested by Notter and Sleicher [11] have been used to represent the eddy diffusivity.

$$\frac{\varepsilon_H}{\nu} = \frac{Ay^{+3}}{\left[1 + \left(\frac{A}{0.011}\right)^2 y^{+2}\right]^{1/2}} \tag{14}$$

Eq. (14) is selected as it apparently follows the power law (y^3) distribution near the wall ($y^+ < 5$) and transforms smoothly to the quadratic order in the buffer layer as suggested by many researchers. The constant (A) in Eq. (14) can be obtained using the assumption {6}, $y^+ \to 0$, $\varepsilon_M = \varepsilon_H$ [16]. In the present analysis, A was assumed to be 7×10^{-4} the value indicated by Churchill [12]. With this value of A, the eddy diffusivity expression can be written as,

$$\frac{\varepsilon_H}{\nu} = \frac{0.0007y^{+3}}{\left[1 + 0.00405y^{+2}\right]^{1/2}} \tag{15}$$

Eq. (15) is found applicable in the region $0 < y^+ < 30$ for fluids of Pr/Sc one or greater, and the law of wall suggests that this equation is universal for all turbulent boundary layers. The integral function of Eq. (13) can be numerically evaluated using Eq. (15) and represented as $F(Pr, y^+)$.

$$F(Pr, y^+) = \int_0^{y^+} \frac{dy^+}{\frac{1}{Pr} + \frac{0.0007y^{+3}}{\left(1 + 0.00405y^{+2}\right)^{1/2}}} \tag{16}$$

Eq. (16) also represents the dimensionless temperature profile for turbulent fluid flow through pipes and tubes at various Pr. Therefore the function corresponding to interfacial distance represents the dimensionless temperature at the interface. Applying the assumption {7} that eddy viscosity controls the transfer process in turbulent core, eddy diffusivity (ε_H) in Eq. (11) can be replaced with eddy viscosity (ε_M). Hence with $\phi = 1$, Eq. (13) simplifies to,

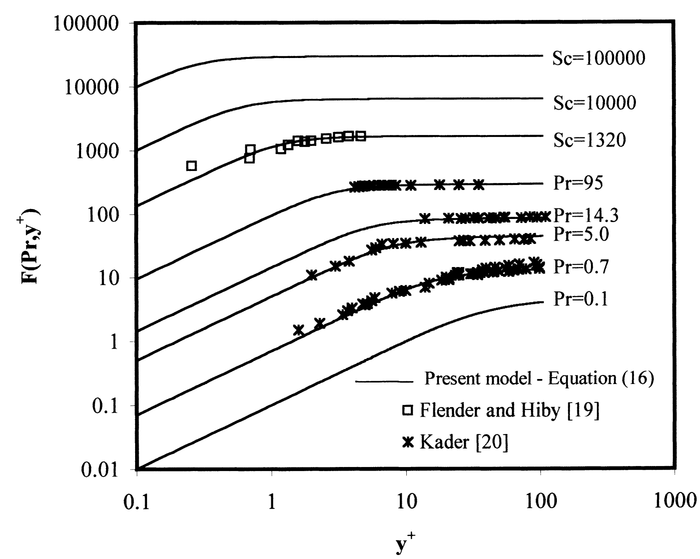

Fig. 2. Graphical representation of $F(Pr, y^+)$ at various Pr/Sc.

Table 2
Details of literature data used for the present analysis

S. No.	Source	Transfer	System	*Re* (range)	*Sc*/*Pr* (range)
1	Harriot and Hamilton [21]	Mass	Benzoic acid–water	10,000–100,000	432–930
			Benzoic acid–glycerol + water	10,000–30,000	1520–97,600
			Benzoic acid–methocel + water	10,000	1810–10,300
2	Linton and Sherwood [22]	Mass	Benzoic acid–water	2450–64,500	950–3320
			Cinnamic acid–water	2400–68,400	1350–3280
			Beta napthol–water	2830–46,700	1740–2460
3	Meyerink and Friedlander [23]	Mass	Benzoic acid–water	4715–22,900	900
			Cinnamic acid–water	8800–22600	900
			Aspirin–water	4800–25,500	850
4	Friend and Metzner [7]	Heat	Air, H_2, N_2, NaOH, water, *n*-butanol, glycol, molasses and corn syrup	10,000	0.46–590
5	Kolar [24]	Heat	Air	8600–94,000	0.7
6	Malina and Sparrow [25]	Heat	Water and oil (chlorinated biphenyl)	13,000–111,000	3.75
7	Morris and Whitman [26]	Heat	Water, straw oil, gas oil and light motor oil	3200–45,000	2–276
8	Sherwood and Petrie [27]	Heat	Water, acetone, kerosene, benzene and *n*-butanol	2610–119,500	2–33

$$St = \frac{\frac{f}{2}}{1 + \sqrt{\frac{f}{2}}\left(T_i^+ - u_i^+\right)} \tag{17}$$

The thickness of interface from the wall is assumed constant ($y^+ = 26$), as recommended in most of the previous works. The velocity at the interface can be found out using the von Karman relation for dimensionless velocity.

$$u^+ = 5\ln\left(y^+\right) - 3.05 \tag{18}$$

Hence Eq. (17) can be written as

$$St = \frac{\frac{f}{2}}{1 + \sqrt{\frac{f}{2}}(F(Pr, 26) - 13.2)} \tag{19}$$

Lin et al. [17] and many other researchers have derived expressions similar to Eq. (17) using material balance approach with the assumptions of constant heat flux, and $Pr_t = 1$.

2.2. Analogy between transfer processes in turbulent pipe flow

The analogy between heat, mass and momentum transfer can be described for turbulent pipe flow by introducing momentum transfer coefficient, similar to film coefficients [18]. The shear stress for turbulent pipe flow can be defined in terms of axial momentum difference per unit volume between the bulk fluid and fluid at the wall.

$$\tau_w = k_m(\rho u_b - \rho u_w) \tag{20}$$

Since the velocity at the wall $u_w = 0$, Eq. (20) reduces to simple form relating momentum transfer coefficient (k_m) with friction factor and bulk velocity.

$$\tau_w = k_m \rho u_b = \left(\frac{f}{2}u_b\right)\rho u_b \tag{21}$$

Similar to Nusselt number for heat transfer or Sherwood number for mass transfer, the momentum transfer coefficient, can be written in dimensionless form, called Fanning number (Fa).

$$Fa = \frac{k_m d}{\nu} = \frac{f}{2}Re \tag{22}$$

Using Eq. (17), the analogy between heat, mass and momentum can be written as,

$$\frac{f}{2} = \frac{Nu}{Re\,Pr}\left[1+\sqrt{\frac{f}{2}}\left(T_{\rm i}^{+}-u_{\rm i}^{+}\right)\right] = \frac{Sh}{Re\,Sc}\left[1+\sqrt{\frac{f}{2}}\left(C_{\rm i}^{+}-u_{\rm i}^{+}\right)\right] = \frac{Fa}{Re}\left[1+\sqrt{\frac{f}{2}}\left(u_{\rm i}^{+}-u_{\rm i}^{+}\right)\right] \quad (23)$$

Eq. (23) appears to confirm the assumption {7}, as otherwise it leads to mathematical inequality. If the assumption {7} is invalid, it yields $\phi \neq 1$ hence the momentum transfer term in Eq. (23) do not reduce to $f/2$. It is found that Eq. (23) is applicable over entire range of Pr/Sc and introduction of Fanning number elucidates the analogy between heat, mass and momentum transfer.

2.3. Analysis and discussion

Eq. (16) can be numerically integrated using Simpson's 3/8th rule and the calculated fully developed radial temperature or concentration profiles at various Pr/Sc are presented in Fig. 2. Eq. (16) is found to represent closely the radial temperature or concentration profiles reported in the literature [19,20]. Based on the reported experimental film coefficients, Eq. (19) was tested with the 618 experimental data points. The data compiled by Friend and Metzner [7] were also included for the present analysis. Table 2 lists the details of literature data used for the present analysis. Fig. 3 compares the experimental values with those predicted by Eq. (19) for various sets of data on turbulent heat and mass transfer in pipes. The predicted values are found to be in good agreement with the literature data with an average absolute error of 10.8% over the range $0.4 \leq Pr/Sc \leq 1 \times 10^5$.

At $Pr = 1$, the function $F(Pr, 26)$ yields a value of 14.0. In turbulent regime, the denominator term in Eq. (19), with $F(1, 26) = 14.0$ can be approximated to unity. Hence Eq. (19) approaches the Reynolds analogy at $Pr = 1$. For practical purpose the function $F(Pr, 26)$ in Eq. (19) can be approximated in terms of

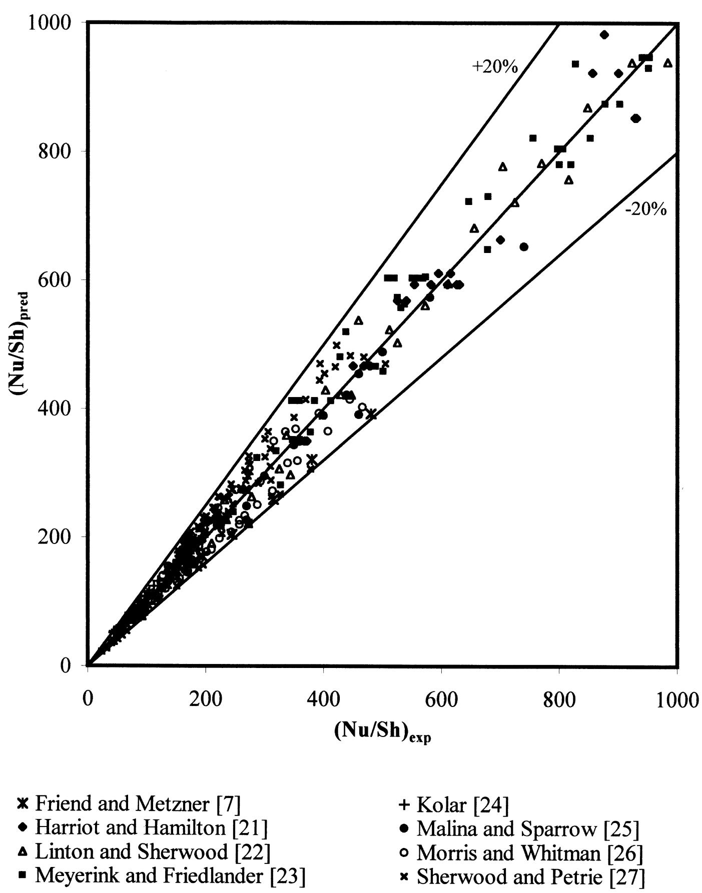

Fig. 3. Comparison of experimental film coefficients with those predicted using Eq. (19).

Table 3
Comparison of error with other equations

Pr/Sc range	0.4–10	10–10^2	10^2–10^3	10^3–10^5	0.4–10^5
Number of data points	286	156	98	78	618
Equation	Abolute error percent average				
Chilton–Colburn	18.2	13.1	23.0	26.7	18.7
Petukhov and Popov [3]	14.5	17.4	15.2	18.9	15.9
Petukhov [4]	14.0	10.7	9.5	8.7	11.8
Sandal et al. [5]	16.7	12.3	13.1	10.3	14.2
Friend–Metzner [7]	10.2	10.5	13.7	10.2	10.8
Gnielinski [8]	13.8	13.0	11.1	12.9	13.1
Pinczcwski and Siderman [9]	19.1	13.1	11.6	14.1	15.8
Present paper, Eq. (19)	11.7	11.3	8.9	8.8	10.8
Present paper, Eq. (25)	12.3	11.4	9.2	10.4	11.3
Present paper, Eq. (26)	11.1	10.6	8.6	8.5	10.2

Prandtl number. Musschenga et al. [18] using extended random surface renewal (ERSR) model, found that the function $F(Pr, y^+)$ is proportional to $Pr^{2/3}$. Fig. 4 shows the dependence of $F(Pr, 26)$ on Prandtl number in logarithmic coordinates. The function appears to be well represented by a linear relation on these coordinates, and can be expressed in simple exponential form as,

$$F(Pr, 26) = 14.0(Pr)^{2/3} \tag{24}$$

Using the above relation, Eq. (19) can be written as,

$$St = \frac{\frac{f}{2}}{1 + \sqrt{\frac{f}{2}}(14.0Pr^{2/3} - 13.2)} \tag{25}$$

Eq. (25) was also found to give good prediction with an average absolute error of 11.3%, which is better than that achieved by most authors. However, Eq. (25) can be simplified as,

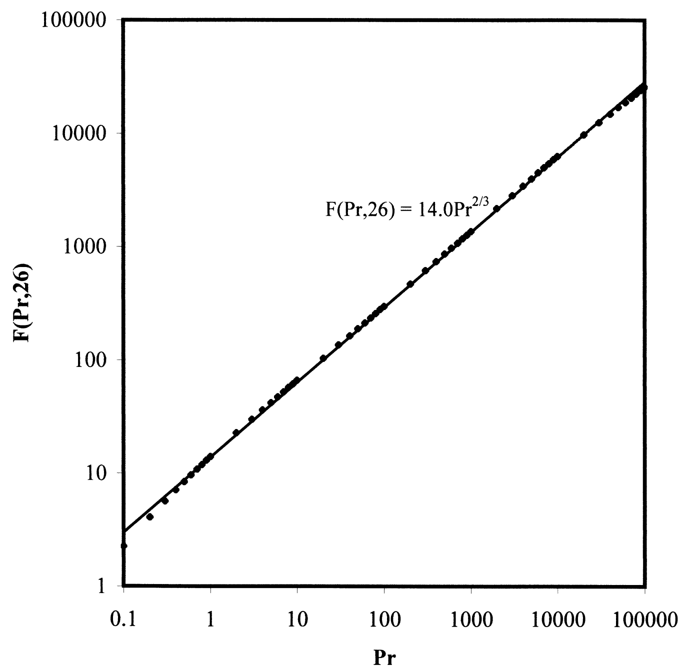

Fig. 4. Evaluation of function $F(Pr, 26)$.

$$St = \frac{\frac{f}{2}}{1.2 + 13.2\sqrt{\frac{f}{2}}(Pr^{2/3} - 1)} \quad (26)$$

Eq. (26) is similar to that of Petukhov [4] and found to give best prediction over entire Pr/Sc range. Table 3 compares the predictability of proposed equations with the previously published relations at various ranges of Pr and Sc. At higher Prandtl number, Eq. (26) can be approximated as,

$$Nu = 0.0757 Re\ Pr^{1/3}\sqrt{\frac{f}{2}} \quad (27)$$

Hence the exponent on friction factor attains a limiting value of 1/2 required by von Karman hypothesis. Eq. (27) also represents the theoretical cube root dependence on Prandtl number, in agreement with the Chilton–Colburn analogy. Using the Blasius friction factor correlation, Eq. (27) reduces to simple equation proposed by Notter and Sleicher [11] for $Pr > 100$,

$$Nu = 0.015 Re^{0.88}\ Pr^{1/3} \quad (28)$$

As mentioned by Friend and Metzner [7], Eq. (28) appears to confirm that at higher Prandtl number the exponent on Reynolds number approaches the value of 0.90.

3. Conclusion

The present analysis seems to confirm that at low Prandtl number, the viscous sub-layer, buffer regime and turbulent core offer resistance to the transfer process, while at higher Prandtl number, the region near the wall offers major resistance. All the transfer operations involving a phase boundary have been developed with resistance in series concept. In a similar manner a model has been developed for the transfer operation during fluid flow through pipe, as it involves solid–fluid transfer process. Eq. (23) validates the assumption that the transfer process depends upon eddy diffusivity in viscous-buffer zone, while in turbulent core it depends upon eddy viscosity. This leads to the conclusion that the resistance in the viscous-buffer zone depends on the characteristics of the solute at the wall (mass transfer) or film adjacent to the wall (heat transfer); while in turbulent core the resistance depends on that of the fluid properties. Therefore, it can be assumed that the viscous-buffer zone represents the wall (solid) side transfer coefficient, while the turbulent core represents the fluid side transfer coefficient.

A new expression for eddy diffusivity has been formulated using the limit $y^+ \to 0, Pr_t = 1$. The present eddy diffusivity expression neglecting the effect of Prandtl number while considering the radial variation of Pr_t is found to represent the transfer process better than the published correlations. Most of the previous models, were based on assumption of $Pr_t = 0.85$. Although this is true in the turbulent core, this can be neglected since only the wall region offers major resistance to the transfer process.

In general, from the data analysis it is observed that the error of prediction is higher in the applicable range of Pr/Sc, since all the equations are based on constant fluid property assumption. In heat transfer applications or at low Pr, this assumption is not applicable, while in case of mass transfer operations or at high Sc the fluid properties approach this assumption. Hence, all the equations should give good prediction at high Sc. But the practical applicability of an equation depends upon its ability to predict transfer coefficients under non-isothermal condition or at low Pr. Among the published correlations, Friend and Metzner equation [7] represents the experimental data better than other equations. In fact it gives best prediction at low Pr/Sc. Petukhov equation [4] also predicts the experimental data with good accuracy. Other correlations predict the experimental data fairly well.

Although Eqs. (19), (25) and (26) can represent the transfer process during flow through pipes, the recommended equation for use is Eq. (26) as it provides a simple and accurate means for predicting transport coefficients over a wide range of data. Such a generalized correlation can be safely applied for design calculations of heat and mass transfer equipments. Though the semi-theoretical correlation presented in this work satisfactorily represents limiting or isothermal coefficient of heat transfer, non-isothermal deviations may also be approximated with the present state of art using Sieder–Tate viscosity correction. This concept can be extended for other solid–fluid transfer processes involving turbulent flow.

References

[1] E.R. Gilliland, T.K. Sherwood, Diffusion of vapours into air streams, Ind. Eng. Chem 26 (1934) 516–523.

[2] A. Dudukovic, V. Milosevic, R. Pjanovic, Gas–solid and gas–liquid mass transfer coefficients, AIChE J. 42 (1996) 269–270.

[3] B.S. Petukhov, V.N. Popov, Theoretical calculation for heat exchange and frictional resistance in turbulent flow in tubes of an incompressible fluid with variable physical properties, High Temperature J. 1 (1963) 69–83.

[4] B.S. Petukhov, Heat transfer and friction in turbulent pipe flow with variable fluid properties, Adv. Heat Transfer 6 (1970) 503–540.

[5] O.C. Sandall, O.T. Hanna, P.R. Mazet, A new theoretical formula for turbulent heat and mass transfer with

gases or liquids in tube flow, Can. J. Chem. Eng 58 (1980) 443–447.

[6] H. Reichardt, Fundamentals of turbulent heat transfer. Nat. Adv. Comm. Aeronaut., Washington, DC, TN-41947, 1955.

[7] W.L. Friend, A.B. Metzner, Turbulent heat transfer inside tubes and the analogy among heat, mass and momentum transfer, AIChE J. 4 (1958) 393–402.

[8] V. Gnielinski, New equations for heat and mass transfer in turbulent flow pipes and ducts, Forsch. Ing.-Wes 41 (1975) 8–16.

[9] W.V. Pinczewski, S. Sideman, A model for mass (heat) transfer in turbulent tube flow. Moderate and high Schmidt (Prandtl) numbers, Chem. Eng. Sci. 29 (1974) 1969–1976.

[10] G.A. Hughmark, Momentum, heat, and mass transfer analogy for turbulent flow in circular pipes, Ind. Eng. Chem. Fundamentals 8 (1969) 31–35.

[11] R.H. Notter, C.A. Sleicher, The eddy diffusivity in the turbulent boundary layer near a wall, Chem. Eng. Sci. 26 (1971) 161–171.

[12] S.W. Churchill, A critique of predictive and correlative models for turbulent flow and convection, Ind. Eng. Chem. Res. 35 (1996) 3122–3140.

[13] R.G. Deissler, C.S. Eian, Analytical and experimental investigation of adiabatic turbulent flow in smooth tubes. Nat. Adv. Comm. Aeronaut., Washington, DC, TN-2629, 1952.

[14] R.G. Deissler, Analysis of turbulent heat transfer, mass transfer and friction in smooth tubes at high Prandtl and Schmidt. Nat. Adv. Comm. Aeronaut., Washington, DC, TN-1210, 1955.

[15] W.M. Kays, M.E. Crawford, in: Convective Heat and Mass Transfer, 3rd ed., McGraw-Hill, New York, 1993, p. 262.

[16] S.L. Lyons, T.J. Hanratty, J.B. McLaughlin, Direct numerical simulations of passive heat transfer in turbulent channel flow, Int. J. Heat and Mass Transfer 34 (1991) 1149–1158.

[17] C.S. Lin, R.W. Moulton, G.L. Putnam, Mass transfer between solid wall and fluid streams, mechanism and eddy distribution relationships in turbulent flow, Ind. Eng. Chem. 45 (1953) 636–640.

[18] E.E. Musschenga, P.J. Hamersma, J.M.H. Fortuin, Momentum, heat and mass transfer in turbulent pipe flow: the extended random surface renewal model, Chem. Eng. Sci. 47 (1992) 4373–4392.

[19] J.F. Flender, J.W. Hiby, Investigation of solid/liquid mass transfer by a photometric method, Ger. Chem. Eng. 4 (1981) 370–379.

[20] B.A. Kader, Temperature and concentration profiles in fully turbulent boundary layers, Int. J. Heat and Mass Transfer 24 (1981) 1541–1544.

[21] P. Harriot, R.M. Hamilton, Solid–liquid mass transfer in turbulent pipe flow, Chem. Eng. Sci. 20 (1965) 1073–1078.

[22] W.H. Linton, T.K. Sherwood, Mass transfer from solid spheres to water in streamline and turbulent flow, Chem. Eng. Prog. 46 (1950) 258–263.

[23] E.S.C. Meyerink, S.K. Friedlander, Diffusion and diffusion controlled reactions in fully developed turbulent pipe flow, Chem. Eng. Sci. 17 (1962) 121–135.

[24] V. Kolar, Heat transfer in turbulent flow of fluids through smooth and rough tubes, Int. J. Heat and Mass Transfer 8 (1965) 639–653.

[25] J.A. Malina, E.M. Sparrow, Variable property, constant-property, and entrance-region heat transfer results for turbulent flow of water and oil in a circular tube, Chem. Eng. Sci. 19 (1964) 953–962.

[26] F.H. Morris, W.G. Whitman, Heat transfer for oils and water in pipes, Ind. Eng. Chem. 20 (1928) 234–240.

[27] T.K. Sherwood, J.M. Petrie, Heat transmission to liquids flowing in pipes, Ind. Eng. Chem. 24 (1932) 736–745.